1 MATLAB Example

Download the ICA MATLAB code from Moodle. Open and run the `ica_image_mix.m` file. The code loads a pair of images, superimposes them, and then unmixes the images. Answer the following questions, and then modify the code to test that your reasoning is correct.

1. Take two greyscale images. Pick one of them and generate an inverted version of it (white becomes black and vice versa). Will this affect the result of ICA when you try to unmix the inverted image with the second image? What happens if you mix the inverted and the original greyscale image?
2. Is ICA insensitive to the choice of correlation created in the mixing matrix? To test this, modify the seed of the random mixing matrix.
3. Will ICA be affected if you modify one of the images by adding a vector of random white noise entries?

2 Whitening

Recall that in ICA, whitening is done by projecting a zero-mean distribution x through the matrix $V = D^{-\frac{1}{2}}E^{\top}$, where E is the matrix consisting of the eigenvectors of the covariance matrix of x, and D is a diagonal matrix composed of the eigenvalues of the corresponding eigenvectors in E.

1. Explain how such a projection whitens the data, i.e., how it ensures that the data, once projected, is uncorrelated and has unit variance.
2. While this projection is done in one time step, show that $z = Vx$ is a stationary point of the iterative learning rule $\Delta V = \gamma(I - zz^{\top})V$.

3 Negentropy

ICA uses a fundamental property of Gaussian distributions to estimate the independent components. This property is that the entropy of a Gaussian distribution is larger than any other distribution with the same mean and variance.

1. For a random variable x and the two associated probability density functions $g(x)$ (corresponding to Gaussian distribution) and $f(x)$ with the same mean and variance, show that the above property is true.

   Hint: Use the fact that the relative entropy $D(f||g)$ of two probability density functions $f(x)$ and $g(x)$ is non-negative, i.e., $D(f||g) = \int_{-\infty}^{\infty} f(x) \ln\left(\frac{f(x)}{g(x)}\right) dx \geq 0$.
2. Show that the negentropy is thus always non-negative and discuss what this means for ICA.